replaced, mixed thoroughly, and the process repeated. It was previously decided to stop at the end of 600 trials. Throughout the work, the ratio of relatively prime pairs to total trials oscillated near the value .6, and at the conclusion, the number of relatively prime pairs was 364, a value surprisingly close to the correct theoretical value 364.8. Of course, an experiment involving, say, 10,000 trials would be more satisfying, but this result is perhaps not without interest.

493 (Algebra). Proposed by ALBERT BABBITT, University of Nebraska.

Determine the coefficients b, c, d of the equation $x^3 + bx^2 + cx + d = 0$ so that they shall be roots of the same equation. [From *Supplemento a Periodico di Matematica.*]

SOLUTION BY H. S. UHLER, Yale University.

Since the coefficient of x^3 is $+1$ and as b, c, d are to be roots, we have the following conditions:

$$-b = b + c + d \quad (1), \qquad c = bc + cd + db \quad (2), \qquad -d = bcd \quad (3).$$

Equation (3) gives

$$d = 0 \quad (4), \qquad \text{or} \qquad bc = -1 \quad (5).$$

Combining (4) with (1) and (2) we get

$$2b + c = 0 \quad (1'), \qquad bc - c = 0 \quad (2').$$

Combining (5) with (1) and (2) we find $2b^2 + bd - 1 = 0$ (1″) and $b^2d - b$ - .

Equations (1′) and (2′) lead at once to the two pairs of values $b = 0$, $c = 0$, and $b = 1$. $c = -2$.

The factors of (2″) give $b = 1$ and $d = (b + 1)^{-1}$.

Substituting $b = 1$ in (5) and (1″) we find respectively $c = -1$ and $d = -1$. Replacing d by $(b + 1)^{-1}$ in (1″) gives

$$2b^3 + 2b^2 - 1 = 0. \tag{6}$$

The discriminant of (6) is positive so that this cubic has two complex roots and one positive real root. These roots may be expressed as

$$b = \tfrac{1}{6}[(46 + 6\sqrt{57})^{1/3} + (46 - 6\sqrt{57})^{1/3}] - \tfrac{1}{3}.$$

The approximate value of the real root, b_1, is

$$b_1 = +0.565{,}197{,}717{,}38.$$

Accordingly, the complex roots are approximately

$$b = \tfrac{1}{2}[-(b_1 + 1) + i\sqrt{(b_1 + 1)(3b_1 - 1)}],$$

or

$$b_2 = -0.782{,}598{,}858{,}69 + i0.521{,}713{,}717{,}94,$$

$$b_3 = -0.782{,}598{,}858{,}69 - i0.521{,}713{,}717{,}94.$$

The corresponding values of c and d may be found by the aid of (5) and $d = (b_1 + 1)^{-1}$, respectively. Finally, collecting all the results

b	c	d
0	0	0
1	-2	0
1	-1	-1
b_1	$-b_1^{-1}$	$(b_1 + 1)^{-1}$
b_2	$-b_2^{-1}$	$(b_2 + 1)^{-1}$
b_3	$-b_3^{-1}$	$(b_3 + 1)^{-1}$

Also solved by J. E. ROWE and the Proposer.

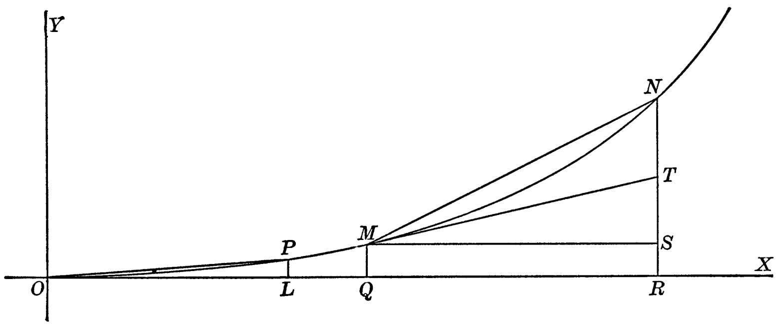
524 (Geometry). Proposed by NORMAN ANNING, Somewhere in France.

Many railways use as "easement curve" the cubic parabola. If points on such a curve are named by their distances measured along the curve from the point of inflection ("flat end")

show that, within the limits of ordinary practice, *i. e.*, for angles so small that the difference between arc and sine is inappreciable, the deflection from tangent at m to set n is $(n - m)(n + 2m)$ times the deflection from tangent at 0 to 1.

Solution by W. J. Thome, Detroit, Michigan.

Let the cubic parabola $y = ax^3$ be constructed, and on it let the points named 0, 1, m, and n be represented by O, P, M, and N respectively. Through these points draw the lines parallel to the axes as shown in the figure. Also the chords OP and MN, and MT, the tangent at M.



Since the angles are so small that $\theta = \sin \theta$, we also have $\theta = \tan \theta$ and $\cos \theta = 1$. Also, an arc of the curve, its chord, and the projection of either on the X-axis are all equal. Hence we have

$$\measuredangle LOP = \tan LOP = \frac{LP}{OL} = \frac{a(OL)^3}{OL} = a(OL)^2 = a(OP)^2 = a(1)^2 = a,$$

$$\measuredangle SMN = \tan SMN = \frac{SN}{MS} = \frac{RN - RS}{QR} = \frac{RN - QM}{OR - OQ} = \frac{a(OR)^3 - a(OQ)^3}{OR - OQ} = \frac{a(ON)^3 - a(OM)^3}{ON - OM}$$

$$= \frac{a(n)^3 - a(m)^3}{n - m} = \frac{a(n^3 - m^3)}{n - m} = a(n^2 + nm + m^2),$$

$$\measuredangle SMT = \tan SMT = \frac{dy}{dx} = 3ax^2 = 3a(OQ)^2 = 3a(OM)^2 = 3am^2,$$

$$\begin{aligned}\measuredangle TMN &= \measuredangle SMN - \measuredangle SMT \\ &= a(n^2 + nm + m^2) - 3am^2 = a(n^2 + nm - 2m^2) \\ &= (n - m)(n + 2m)a \\ &= (n - m)(n + 2m)\measuredangle LOP.\end{aligned}$$

525 (Geometry). Proposed by C. N. SCHMALL, New York City.

Given a quadrant of a circle AOB, where OA and OB are bounding radii, and a semicircle ACO having OA as a diameter and lying on the same side as the quadrant. Describe a circle which shall touch the two arcs and the radius OB.

I. Solution by W. J. Thome, Detroit, Mich.

Suppose the problem solved and the figure constructed.

Let O and D be the centers of the two given circles and let E be the center of the required circle. Draw DE. Draw OE and extend it until it intersects the given quadrant at H. Through E draw EF and EG perpendicular to OA and to OB, respectively.

The problem may be considered solved if we can obtain a value of EG, the radius of the required circle. Let $OA = 2R$, $OD = R$, and $EG = r$. Then $OE = 2R - r$, $DE = R + r$, and $DF = R - r$. Now $\overline{OE}^2 - \overline{EG}^2 = \overline{DE}^2 - \overline{DF}^2$, since $OG^2 = FE^2$, or